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RESEARCH ARTICLE - HYDROLOGY

Joint frequency analysis of river flow rate and suspended sediment load using conditional density of copula functions

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Abstract
In this study, the river flow rate and suspended sediment load at Deh Molla hydrometric station in Zohreh River in the period of 1983–2018 were used for copula-based joint frequency. The Frank copula was selected as the best copula. The joint frequency analysis of the studied variables was performed based on the conditional density of the copula functions, which leads to the conditional occurrence probability of the variables. The results showed that with increasing river flow rate and suspended sediment load, the joint return period increases. Also, the joint return period for the "and" state is longer than the joint return period for the "or" state. When the river flow rate exceeds the threshold of 100 m^3/s and suspended sediment load exceeds the threshold of 2.7×10^4 ton/day, the joint return periods for the "and" state and for the "or" state, are about 11 and 4 years, respectively.

Keywords Bivariate analysis · Conditional density · Joint return period · Rotated copulas · Simulation

Abbreviations

ICA	Imperialist Competitive Algorithm
GWO	Gray Wolf Optimization Algorithm
EA	Election Algorithm
RMSE	Root Mean Square Error
NSE	Nash–Sutcliffe
R^2	Coefficient of Determination
SSC	Suspended Sediment Concentration
AR	Autoregressive
NRMSE	Normalized Root Mean Square Error
IFM	Inference Functions for Margins

List of symbols

Q_w	River flow rate (m^3/s)
Q_s	Suspended sediment load (ton/day)
X_m	Measured data
X_p	Predicted data
$\overline{X}_m$	Mean of the measured data
$\overline{X}_p$	Mean of the predicted data
$X_{m_{\max}}$	Maximum value of the measured data
$X_{m_{\min}}$	Minimum value of the measured data
$F_X(x)$	Distribution of arbitrary margin of x
$F_Y(y)$	Distribution of arbitrary margin of y
C	Copula
$F_{X,Y}(x, y)$	Joint distribution function
$f_Y(y)$	Density functions of $F_Y(y)$
$f_X(x)$	Density functions of $F_X(x)$
Φ	Generator function
τ	Kendall's tau
Θ	Copula parameter
$\ln L_C$	Log likelihood function
JRP	Joint Return Period
CP	Conditional Probability

Introduction

Accurate estimation of sediment load transferred by rivers is one of the important aspects of river management, irrigation and drainage networks and water resources

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projects. Suspended sediment load is the major part of the total sediment load, which due to its negative effects on water projects, accurate estimation of the suspended sediment load is particular importance. The first sampling of suspended sediment loads in the Mississippi River was performed in 1845 and then expanded worldwide (Miraboulghasemi and Morid 1997). In general, there are many methods for estimating the suspended sediment load in rivers, which are classified into two main categories based on hydraulic-dynamic laws and hydrological methods. One of the most important hydrological methods in estimating the suspended sediment load in hydrometric stations is the relationship between the suspended sediment load data and the corresponding river flow rate data, which is determined by the sediment rating curve. The equation of the sediment rating curve is as follows:

$$Q_s = a\,Q_w^b, \tag{1}$$

where Q_w is the corresponding river flow rate (usually in m^3/s), Q_s is the suspended sediment load (usually in ton/day), and a and b are the coefficients of the equation.

Horowitz (2003) used sediment rating curve to predict suspended sediment load in the Mississippi River. The results showed that in periods of 20 years or more, errors of less than one percent can be obtained using a sediment rating curve based on data spanning the entire period. Blanco et al. (2010) analyzed the variations of suspended sediment load at different time scales (storm, seasonal, monthly, and annual) over a three-year period in northwestern Spain using the statistical relationship between river flow rate and suspended sediment concentration. Their results showed that sediment rating curves do not have an acceptable efficiency in estimating the suspended sediment concentration (especially in flood events). The phenomenon of erosion and sediment transport is one of the most complex hydrodynamic problems that due to the influence of various parameters, it is not easy to simulate its behavior. The complexity of sediment behavior has led to the introduction of new technologies.

Hang and Suetsugi (2013) evaluated the efficiency of an artificial neural network in estimating the suspended sediment load of a Tonle Sap River basin in Cambodia. The results of their research indicate the acceptable performance of the artificial neural network in this field. Emami et al. (2021) used three optimization meta-heuristic algorithms including the Imperialist Competitive Algorithm (ICA), the Gray Wolf Optimization Algorithm (GWO), and the Election Algorithm (EA) to predict the suspended sediment load of the Zarrineh Rood River. To evaluate the performance of the proposed algorithms, they used three statistics including root mean square error (RMSE), Nash–Sutcliffe (NSE), and coefficient of determination (R^2). Their results showed that GWO algorithm with $R^2 = 0.96$, RMSE = 228.86 ton/day and NSE = 0.74 has higher accuracy compared to the other algorithms.

In the researches, different methods have been used to study the suspended sediment load. One of the most recent proposed method for the bivariate analysis of suspended sediment load and river flow rate is copula functions, that proposed by Sklar (1959). According to Sklar theorem, any joint multivariate distribution can be expressed using a copula function and univariate marginal distributions (Sklar 1959). These functions were first used in hydrological studies by De Michele and Salvadori (2003) for frequency analysis of precipitation. The concept of copula functions was then rapidly applied in various fields of hydrology. Among the researches that have been done in hydrological studies using copula functions can be named as creating a bivariate model to describing the intensity and duration of storms (Salvadori and De Michele 2004; Wang et al. 2010; Bushra et al. 2019), analysis of drought characteristics (Laux et al. 2009; Wong et al. 2010; Ayantobo et al. 2018; Hui-Mean et al. 2019), bivariate analysis of rainfall (Pham et al. 2016; Nazeri Tahroudi et al. 2021; Tanim et al. 2021), establishing the relationship between peak flow and flood volume (Favre et al. 2004; Zhong et al. 2020; Luo et al. 2021) and so on.

Bezak et al. (2014) used one hydrometric station from Slovenia and five hydrometric stations from the United States to trivariate analysis of flood peak flow, flood volume and suspended sediment concentration. They selected the best copula function using statistical tests. Their results indicated that the Gumbel-Hougaard copula is the best copula function. They also calculated the initial joint return periods in "or" state and the secondary return periods of Kendall, and a comparison was made between the selected copula functions. Their results showed that the copula functions are useful mathematical tools that can be used to simulate the studied variables. Bezak et al. (2017) also examined a new method for estimating suspended sediment load based on river flow rate and precipitation data using the copula function in two case studies (Kuzlovec River and Mura River). Also, the estimated suspended sediment load using copula functions was compared with different regression models. Their results showed that the copula functions were more consistent with the measured data than the other studied methods.

Peng et al. (2019) used the bivariate copula functions to determine the structures of daily suspended sediment concentration (SSC) dependence at the Pingshan station in the Jinsha River. Also, to reduce the difficulty of daily SSC marginal distributions, the measured daily SSC data were normalized using the normal quantile transform method and compared with the autoregressive (AR) model. The results showed that the proposed method can better maintain the statistical properties of the measured daily SSC data with higher accuracy. In addition, the nonlinear correlation of estimated SSC with the proposed method was better than the AR model. In another study, Peng et al. (2020) studied the multivariate frequency analysis of extreme sediment events in the Jinsha River in China by considering the joint behavior of suspended sediment concentration (SSC), flood peak flow and flood volume using copula functions. Uncertainties related to copula-based simulation were also investigated. The results showed that the copula-based multivariate method gives a more comprehensive assessment of the extreme SSC than the univariate frequency analysis. The uncertainty of copula-based simulation decreases with longer statistical observation period and increases with longer return period. The range of uncertainty of the most likely design quartiles varies with different copula, and the range of uncertainty for the best copula is not the smallest.

Keihani et al. (2021) investigated the multivariate frequency analysis between maximum annual flood discharge, suspended sediment load and bed sediment load at Sira hydrometric station in Karaj River with different copula functions. The results showed that the best copula functions in analyzing the dependence between the variables of flood discharge-suspended sediment load, flood discharge-bed sediment load and suspended sediment load-bed sediment load are Tawn, Shih-Louis and Gaussian, respectively. The results also showed that according to the "and" scenario for the joint return period, the multivariate design values of the suspended sediment load and the bed sediment load are smaller than the univariate values. Thus, ignoring the correlation between suspended sediment load, bed sediment load, and flood discharge may significantly overestimate the real sediment load, as a result, the joint occurrence probability is estimated more.

According to the researches, the estimation of suspended sediment load using copula functions has been less considered. Copula functions are one of the newest methods for multivariate analysis of sediment transport. Therefore, in order to reduce the damage caused by sedimentation and the possibility of optimal design of hydro-structures, bivariate analysis of river flow rate-suspended sediment load using copula functions is necessary and it can lead to the acquisition of valuable information in sediment transport planning. Also, in conventional univariate or bivariate frequency analysis, the existence of the same marginal distribution is one of the main points of the study, which is one of the limitations in this field. On the other hand, the study of the bivariate behavior of the river flow rate and the corresponding suspended sediment load depends on the joint cumulative distribution function, which can be implemented with the help of copula functions. Accordingly, in this study, by studying the bivariate behavior of the river flow rate and corresponding suspended sediment load, the joint frequency analysis was performed using conditional density of copula functions. Due to the dependence of suspended sediment load on river flow rate, a copula-based conditional density approach was used to estimate the conditional occurrence probability. In order to fully cover the dependence in different directions, the rotated states of copula functions were investigated. This study focuses on the copula functions and its conditional density seeks to bivariate simulation of suspended sediment load- river flow rate that can provide typical curves to estimate suspended sediment load affected by the river flow rate.

Materials and methods

Case study

The Zohreh River basin is located in the southern part of the Zagros Mountains. There are 26 hydrometric stations in the Zohreh River basin that in this study, the data of Deh Molla hydrometric station, which is the last hydrometric station before the river evacuation into the Persian Gulf, has been used. Deh Molla hydrometric station is located in the geographical range of 49 degrees and 40 min of longitude and 30 degrees and 30 min of latitude. The area of the upstream basin of this hydrometric station is 13034.4 km^2 and its height above mean sea level is estimated at 32 m. In order to investigate the accuracy of the copula-based model in joint frequency analysis of river flow rate-suspended sediment load of the Zohreh River, 578 data were used in the period of 1983–2018. Figure 1 shows the location of the Deh Molla hydrometric station in Zohre River. The box plot of the studied data (river flow rate and suspended sediment load) at Deh Molla hydrometric station in Zohreh River were presented in Figs. 2 and 3. Table 1 also shows the statistical characteristics of the studied data.

According to Table 1 and Figs. 2 and 3, the minimum river flow rate is 2.1 m^3/s, the maximum river flow rate is 1139 m^3/s, and the average river flow rate is 95.56 m^3/s. Also, the minimum, maximum and average of suspended sediment load during the studied period are 8.16, 1,278,012.67, and 26,385.05 ton/day, respectively.

Marginal distribution

In order to fit the marginal distribution on the river flow rate and suspended sediment load data, 11 marginal distributions such as normal, lognormal, exponential, gamma, generalized extreme value, logistics, log logistics, Rayleigh, Nakagami, generalized Pareto and Weibull were considered (Forbes et al. 2011). By choosing the type of marginal distribution, a better estimate of the occurrence of extremely natural phenomena with a low probability can be obtained. From the various marginal distributions,

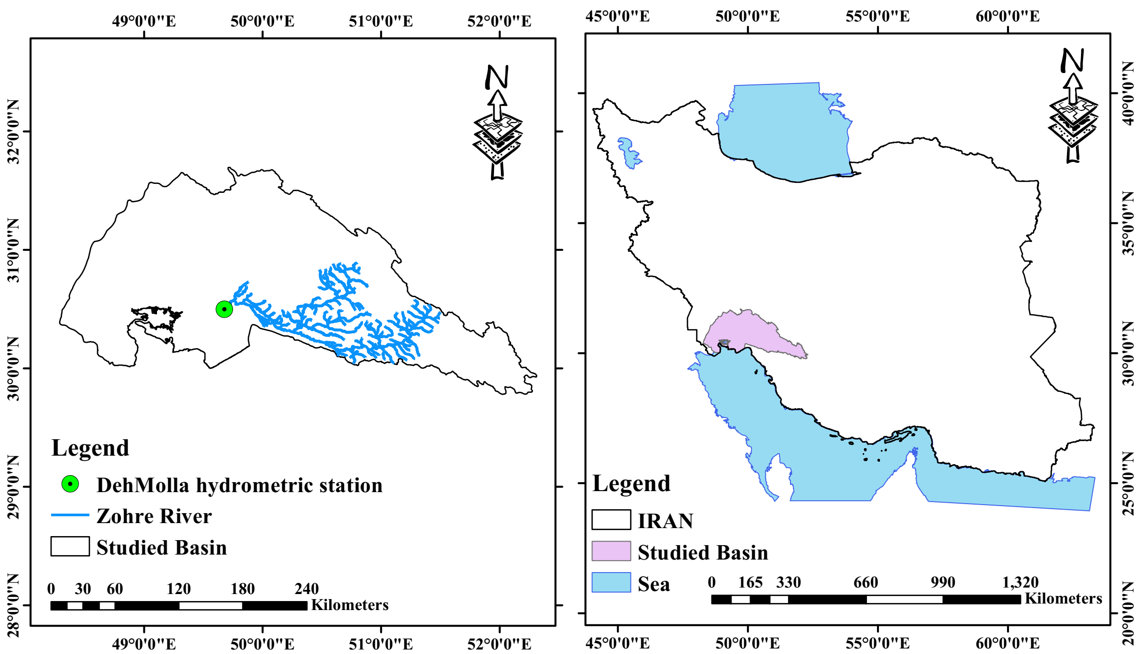


Fig. 1 Location of Deh Molla hydrometric station in Zohreh River

one should choose the best one that has the best fit with the measured data. The normalized root mean square error (NRMSE), BIAS, Nash–Sutcliffe (NSE), coefficient of determination (R^2) and statistics were used to select the best marginal distribution. Whichever distribution has the highest NSE and the lowest NRMSE (Nash and Sutcliffe 1970), the BIAS is close to zero and R^2 is close to one, is selected as the best marginal distribution for each river flow rate and the suspended sediment load distribution. These statistics are calculated as Eqs. 2, 3, 4, and 5:

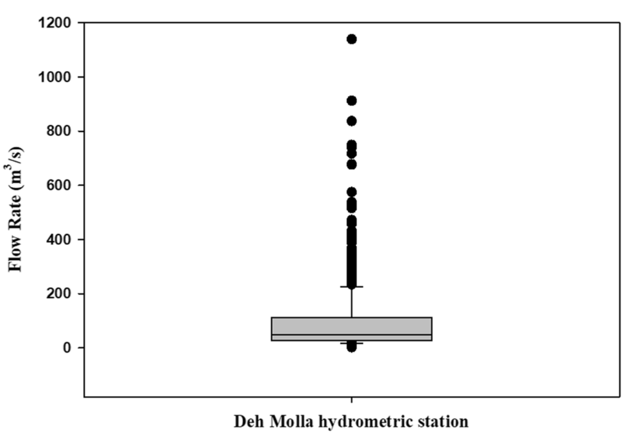


Fig. 2 Box plot of the river flow rate at the Deh Molla hydrometric station in Zohreh River in the period 1983–2018

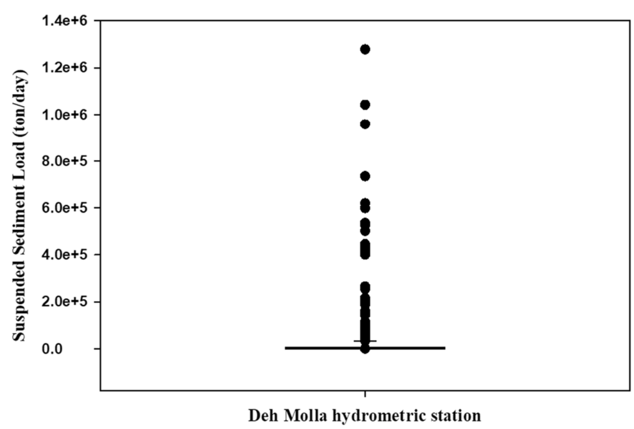


Fig. 3 Box plot of the suspended sediment load at the Deh Molla hydrometric station in Zohreh River in the period 1983–2018

$$\mathrm{NRMSE} = 100 \times \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{n}\left(X_p - X_m\right)^2}}{X_{m_{\max}} - X_{m_{\min}}}, \tag{2}$$

$$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{n}\left(X_p - X_m\right)^2}{\sum_{i=1}^{n}\left(X_m - \overline{X}_m\right)^2}, \tag{3}$$

$$\mathrm{BIAS} = \frac{1}{N}\sum_{i=1}^{n}\left(X_p - X_m\right), \tag{4}$$

Table 1 Statistical characteristics of the studied data

Parameter	Minimum	Maximum	Average	STD
River flow rate $\left(\frac{m^3}{s}\right)$	2.1	1139	95.56	129.9
Suspended sediment load $\left(\frac{\text{ton}}{\text{day}}\right)$	8.16	1,278,012.67	26,385.05	109,456.31

$$R^2 = \left(\frac{\sum_{i=1}^{n} \left(X_m - \overline{X}_m\right)\left(X_p - \overline{X}_p\right)}{\sqrt{\sum_{i=1}^{n} \left(X_m - \overline{X}_m\right)^2} \sqrt{\sum_{i=1}^{n} \left(X_p - \overline{X}_p\right)^2}} \right)^2, \tag{5}$$

where n is the number of data, X_m is the measured data, X_p is the predicted data, $\overline{X}_m$ is the mean of the measured data, $\overline{X}_p$ is the mean of the predicted data, $X_{m_{\max}}$ and $X_{m_{\min}}$ are the maximum and minimum values of the measured data, respectively (Nash and Sutcliffe 1970; Tahroudi et al. 2022).

Copula functions

The copula is a multivariate distribution function that joints univariate marginal distributions to investigate the joint occurrence probability (Nelsen 2006). The basic information of copula function was first introduced by Sklar (1959). In a bivariate method, if two random variables x and y follow the distribution of arbitrary margins $F_X(x)$ and $F_Y(y)$, then a copula C combines these two margins that joint distribution function of $F_{X,Y}(x, y)$ is as follows (Nelsen 2006):

$$F_{X.Y}(x.y) = C\left(F_X(x).F_Y(y)\right). \tag{6}$$

If the marginal distributions $F_X(x)$ and $F_Y(y)$ are continuous, C is unique, and the density function of the joint occurrence probability will be as follows:

$$f_{X.Y}(x.y) = c\left(F_X(x).F_Y(y)\right)f_X(x)f_Y(y), \tag{7}$$

where $f_Y(y)$ and $f_X(x)$ are the density functions of $F_X(x)$ and $F_Y(y)$ and c is the density function of C, that is calculated as follows (Dastourani and Nazeri Tahroudi 2022):

$$c(u.v) = \frac{\partial^2 \mathrm{C}(u.v)}{\partial u \partial v}. \tag{8}$$

In above equation, u and v are univariate cumulative distribution functions. There are many families of copulas such as elliptical family (normal and t); Archimedean family (Clayton, Gamble, Frank, and Ali-Mikhail-Haq); extreme values family (Gamble, Hassler-Rees, Galambos, Tawn, and t-EV), and other families (Plackett and Farlie-Gumbel-Morgenstern) (Joe 1997; Nelsen 2006; Capéraà et al. 2000; Fang et al. 2002; Abdous et al. 2004). The Archimedean copulas and extreme values (EVs) are more commonly used in hydrological studies.

For an Archimedean copula, there is a generator function φ as follows:

$$C(u.v) = \varphi^{-1}\{\varphi(u) + \varphi(v)\}. \tag{9}$$

The generator function is continuous and completely descending, defined in the range [0,1] and $\varphi\,(1) = 0$.

If $\varphi\,(1) = -\ln t$, then $C(u.v) = uv$, in other words, $(u.v)$ are independent (Dastourani and Nazeri Tahroudi 2022).

Nelsen (2006) mentioned some important families of an Archimedean copula along with their generator function, range of parameters and some special and limiting cases. If a copula function can be classified from the extreme values of copula family if there is a convex function called the Pickands dependence function, A: $[0.1] \to \left[\frac{1}{2}.1\right]$ such that for any $t \in [0, 1]$ satisfy the conditions $\mathrm{A}(0) = \mathrm{A}(1) = 1$ and $\max\{t, 1-t\} \le \mathrm{A}(t) \le 1$. In this case, extreme values copula for all $(u.v) \in [0.1]^2$ can be defined as follows (Nelsen 2006):

$$C(u.v) = \exp\left[\log(\mathrm{uv})A\left(\frac{\log v}{\log(\mathrm{uv})}\right)\right], \tag{10}$$

for all $(u.v) \in I^2$. Specifically, if $\mathrm{A}(t) = 1$ then $(u.v)$ are independent, and if $\mathrm{A}(t) = \max\{t, 1-t\}$ then $(u.v)$ are completely dependent (or comonotonic). Conversely, with respect to the bivariate extreme values of copula C, the dependent Pickands function (A) with $t \in [0.1]$ is as follows:

$$A(t) = -\ln C\left(e^{-(1-t)}.e^{t}\right). \tag{11}$$

In this study, some univariate copulas, including Ali-Mikhail-Haq, Clayton, Farlie-Gumbel-Morgenstern, Galambos, Frank, Plackett, and Gumbel-Hougaard were considered to determine the joint occurrence probability of river flow rate and suspended sediment load at the Deh Molla station in the Zohreh River.

Rotated copulas

Many copulas, such as Gumbel, Clayton, and etc., cannot display negative tail dependencies. When the bivariate random variable has a negative dependence, these copulas will not fit. These copulas may "rotate" and be re-applied. A complete review of rotated copulas can be found in Cach (2006) and Luo (2011). A rotated copula can be obtained from a non-rotated copula as below:

For the 180° rotate of a copula, Cach (2006) defined $\overline{u} = 1 - u$ and $\overline{v} = 1 - v$ and proposed the copulas $\overline{u}$ and $\overline{v}$ as below:

$$C^{--}(u.v) = u + v - 1 + C(1 - u.1 - v), \tag{12}$$

where $c^{--}(u.v) = c(1 - u.1 - v)$. Cach (2006) called C^{--} a survival copula.

For the 90° rotate copula, Cach (2006) proposed the following equation:

$$C^{-+}(u.v) = u - C(1 - u.v), \tag{13}$$

where $c^{-+}(u.v) = c(1 - u.v)$ is the copula density C^{-+}.

For the 270° rotate copula, Cach (2006) concluded that:

$$C^{+-}(u.v) = u - C(u.1 - v), \tag{14}$$

where its density is $c^{+-}(u.v) = c(u.1 - v)$.

Dependency analysis

The problem of measuring the dependence between the studied variables is a major problem in multivariate extreme modeling. Traditional measurement of dependence using Pearson's linear correlation coefficient ρ has some problems for bivariate distributions. This coefficient contains only one type of linear dependence and may not provide accurate results when there is no heavy-tailed the data. Therefore, it may not be appropriate to describing extreme (Joe 1997). To overcome Pearson's ρ shortcomings, nonparametric criteria such as rank correlations Spearman ρ and Kendall's τ have been considered (Joe 1997; Nelsen 2006). The mentioned criteria are always present and model different types of dependencies (Salvadori et al. 2007). The Kendall's tau correlation has been selected as the best coefficient for examining correlation between the variables in the most studies in the field of copula functions (De Michele and Salvadori 2003; Khashei-Siuki et al. 2021; Tahroudi et al. 2022; Nazeri Tahroudi et al. 2021 and 2022; Dastourani and Nazeri Tahroudi 2022). The Kendall's tau is not identical in size because its basic logic and computational formulas are quite different. Also, the Kendall's tau correlation can interpret its value as a direct measurement of coordinated and contrasting pairs (Nelsen 2006; Xu et al. 2010).

Kendall's τ is a rank correlation coefficient that used to determine the measurement of dependence between random variables. The population version of Kendall's τ is defined as the difference between the probability of concordance and the probability of discordance:

$$\tau = P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) > 0\right] - P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) < 0\right], \tag{15}$$

where $P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) < 0\right]$ and $P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) > 0\right]$ indicate the probability of discordance and the probability concordance, respectively (Nelsen 2006). For two pairs $\left(x_i.y_i\right)$ and $\left(x_j.y_j\right)$, they match when $\left(x_i - x_j\right)\left(y_i - y_j\right) > 0$, and when $\left(x_i - x_j\right)\left(y_i - y_j\right) < 0$ do not match. For a random sample of n bivariate variables, $\left(x_1.y_1\right).\left(x_2.y_2\right).\dots.\left(x_n.y_n\right)$, the sample version of Kendall's τ can be estimated as follows:

$$\tau = \binom{n}{2}^{-1} \sum_{1 \le i < j \le n} \operatorname{sgn}[(x_i - x_j)\left(y_i - y_j\right)], \tag{16}$$

where $i, j = 1,2,\ldots,n$

$$\operatorname{sgn}(\Psi) = \begin{cases} 1 & \text{if } \Psi > 0 \\ 0 & \text{if } \Psi = 0 \\ -1 & \text{if } \Psi < 0 \end{cases}. \tag{17}$$

The choice of copula depends on the range of dependency levels that the copula can describe and can be determined by Kendall's τ (Nelsen 2006; Salvadori et al. 2007). The studied

Table 2 Univariate copulas and their dependence parameter range (Nelsen 2006)

Copula	C (u,v)	Range of Θ
Ali-Mikhail-Haq	$\frac{uv}{1-\theta(1-u)(1-v)}$	$-1 \le \theta \le 1$
Clayton	$\left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta}$	$\theta \ge 0$
Frank	$-\frac{1}{\theta}\ln\left[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$\theta \neq 0$
Galambos	$uvexp\left\{ \left[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}\right]^{-\frac{1}{\theta}} \right\}$	$\theta \ge 0$
Gumbel-Hougaard	$exp\left\{ -\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{1}{\theta}} \right\}$	$\theta \ge 1$
Plackett	$\frac{1}{2}\frac{1}{\theta-1}\left\{ 1 + (\theta-1)(u+v) - \left[(1+(\theta-1)(u+v))^2 - 4\theta(\theta-1)uv\right]^{\frac{1}{2}} \right\}$	$\theta \ge 0$
Farlie-Gumbel-Morgenstern	$uv[1 + \theta(1-u)(1-v)]$	$-1 \le \theta \le 1$

copula functions and their range of their dependence parameter are shown in Table 2. The dependence parameter (Θ) is used to measure the relationship between u and v. In this study, in order to determine the dependence parameter (Θ) of copula, the inference function for margins (IFM) was used which is one of the most widely used methods for estimating copula parameters (Joe 1997).

In the bivariate method, the two correlated random variables X and Y are distributed as functions $f_X(x;\alpha_1.\alpha_2.\ldots.\alpha_i)$ and $f_Y(y;\beta_1.\beta_2.\ldots.\beta_r)$, in which $\alpha_1.\alpha_2.\ldots.\alpha_i$ are the parameters $f_X(x)$ and $\beta_1.\beta_2.\ldots.\beta_r$ are the parameters $f_Y(y)$. The number of parameters is determined based on the type of selected margin. To estimate the marginal distribution and copula parameters for n pairs of independent observations, the log-likelihood function for X and Y, which are $\ln L_X(x;\alpha_1.\alpha_2.\ldots.\alpha_i)$ and 1 $\ln L_Y(y;\beta_1.\beta_2.\ldots.\beta_r)$ are maximized separately. The log-likelihood function for the joint probability density function $f_{X.Y}(x.y)$ is expressed as Eq. 18:

$$\ln L(x.y;\hat{\alpha}_1.\hat{\alpha}_2.\ldots.\hat{\alpha}_i.\hat{\beta}_1.\hat{\beta}_2.\ldots.\hat{\beta}_r.\theta) = \ln L_C(x.y;F_X(x).F_Y(y).\theta) + \ln L_X(x;\alpha_1.\alpha_2.\ldots.\alpha_i) + \ln L_Y(y;\beta_1.\beta_2.\ldots.\beta_r), \tag{18}$$

where $\ln L_C$ is the log-likelihood function corresponding to the density function of the copulas. By replacing the estimated parameters of the univariate marginal distributions $\alpha_1.\alpha_2.\ldots.\alpha_i$ and $\beta_1.\beta_2.\ldots.\beta_r$ and maximizing the log-likelihood function of $\ln L_C$, the copula parameter (Θ) is obtained (Shiau 2006).

Empirical copulas

For complex multivariate statistical analyzes as well as in evaluating the best fit of parametric copula functions, the most common method is empirical copulas. For example, with size n, the d-dimension empirical copula C_n is as Eq. 19 (Nelsen 2006):

$$C_n\left(\frac{k_1}{n}.\frac{k_2}{n}.\ldots.\frac{k_3}{n}\right) = \frac{a}{n}. \tag{19}$$

In the above equation, a is equal to the number of observations $(x_1.\ldots.x_d)$ that satisfy the condition $x_1 \le x_{1(k_1)}.\ldots.x_d \le x_{d(k_d)}$ in which $x_{1(k_1)}.\ldots.x_d$ with $1 \le k_1.\ldots.k_d \le n$ are sample sequence statistics.

Finally, the results of each copula were compared with the results of the empirical copula and any copula whose their values were closer to the values of the empirical copula was selected as the best copula. NRMSE (Eq. 2), NSE (Eq. 3), BIAS (Eq. 4), and R^2 (Eq. 5) were used to select the best copula function.

Joint return period

In this study, the bivariate frequency analysis was used to evaluate the risk of joint correlated random variables. Bivariate frequency analysis can be defined using the joint return period (JRP) (Yue and Rasmussen 2002). In general, the return period is divided into two states "or" and "and":

A. "or" state: is the joint return period in which one of the random variables X or Y has exceeded the threshold of x or y. In other words, it is $X > x$ or $Y > y$ and is denoted by T_{XY}.
B. "and" state: is the joint return period in which the random variables X and Y both exceed the thresholds of x and y. In other words, it is $X > x$ and $Y > y$ and is denoted by $T\prime_{XY}$.

The joint return period can be defined with the help of appropriate copula functions and will be in the form of Eqs. 20 and 21 (Reddy and Ganguli 2012):

$$T_{XY} = \frac{1}{P(X > x \,\text{or}\, Y > y)} = \frac{1}{1 - F_{X.Y}(x.y)} = \frac{1}{1 - C\left(F_X(x).F_Y(y)\right)}, \tag{20}$$

$$T'_{XY} = \frac{1}{P(X > x \,\text{and}\, Y > y)} = \frac{1}{1 - F_X(x) - F_Y(y) + F_{X.Y}(x.y)} = \frac{1}{1 - F_X(x) - F_Y(y) + C\left(F_X(x).F_Y(y)\right)}. \tag{21}$$

In the above equations, the values of x and y are the same as the risk threshold for random variables, and C is the best copula function. *F(x)* and *F(y)* are the values of the best marginal distribution. Therefore, joint return periods with univariate marginal distribution and copula function can be easily calculated.

Conditional probability

In water resources management, recognizing that the river flow rate and suspended sediment load are greater than a certain threshold is part of management planning. The conditional probability (CP) can be easily investigated using the copula function and bivariate distribution. The conditional probability makes it possible to calculate the probability distribution of X if the Y exceeds a certain threshold (Eq. 22) or is the probability distribution of Y if the X exceeds a certain threshold (Eq. 23) (Shiau 2006):

$$P(X \le x | Y \ge y\prime) = \frac{F_X(x) - F_{YX}(y\prime.x)}{1 - F_Y(y\prime)} = \frac{F_X(x) - C\left(F_Y(y\prime).F_X(x)\right)}{1 - F_Y(y\prime)}, \tag{22}$$

Preparing the river flow rate (m^3/s) and suspended sediment load (ton/day) data

↓

Investigating the correlation between the variables of river flow rate (m^3/s) and suspended sediment load (ton/day)

↓

Selecting the marginal distribution of the studied variables based on NRMSE (%), NSE, BIAS and R^2 statistics

↓

Selecting the best copula function appropriate to the studied variables

↓

Estimating the conditional probability of the studied variables

↓

Calculating the joint return period and comparison with univariate return period results

Fig. 4 Flowchart of the proposed methodology

$$P(Y \leq y|X \geq x\prime) = \frac{F_Y(y) - F_{XY}(x\prime.y)}{1 - F_X(x\prime)} = \frac{F_Y(y) - C\left(F_X(x\prime).F_Y(y)\right)}{1 - F_X(x\prime)}. \tag{23}$$

The confidence intervals are calculated as below:

Upper limit:

$$x_{p,U} = x_p + z_{1-\alpha/2}.\mathrm{SE}[x_p]. \tag{24}$$

Lower limit:

$$x_{p,L} = x_p + z_{\alpha/2}.\mathrm{SE}[x_p] = x_p - z_{1-\alpha/2}.\mathrm{SE}[x_p], \tag{25}$$

where x_p is quantile estimate with probability of exceedance of p, SE[x_p] is its standard error (Sandercock 2015).

The flowchart of the proposed methodology is presented in Fig. 4. In this study, considering the joint frequency analysis of the river flow rate and suspended sediment load, the conditional density of copula functions and common copulas and their rotated states have been used. The most important purpose of this study is to compare the return period in univariate and bivariate methods.

Results

Select the best copula function

In this study, common distribution functions such as normal, lognormal, exponential, gamma, generalized extreme value, logistics, log logistics, Rayleigh, Nakagami, generalized Pareto and Weibull were used to fit the river flow rate and suspended sediment load data. Then, using the normalized root mean square error (NRMSE), Nash–Sutcliffe (NSE), coefficient of determination (R^2) and BIAS, the best marginal distribution was selected. The marginal distribution with the lowest NRMSE, the highest NSE, as well as R^2 close to one and BIAS close to zero was selected as the best marginal distribution function. The generalized extreme value (GEV) was selected as the best marginal distribution for each of the variables. The results are presented in Table 3.

Since copula-based model is required the existence of correlations in the studied data, in this study, Kendall's tau correlation coefficient was used to investigate the correlation between river flow rate and suspended sediment load data. Kendall's tau correlation coefficient was 0.66, which indicates a positive and significant relationship between river flow rate and suspended sediment load data. This degree of dependence between the two variables indicates the need to use copula functions to obtain a joint distribution between the river flow rate and the suspended sediment load data. In order to use each of the copulas, the first must be compare the Kendall's tau correlation coefficient between the studied variables with the allowable range of this coefficient for each copula and then, if it is in the allowable range, simulation were done. The Ali-Mikhail-Haq copula is suitable for weak dependence ($-0.1807 < \tau < 0.3333$) and due to the non-inclusion of the Kendall's tau between the studied variables in the allowable range, this copula was excluded from the simulation. In various studies in the field of copula functions, the existence of correlations between the variables is a prerequisite for implementing a copula-based model. In different studies (De Michele and Salvadori 2003; Khashei-Siuki et al. 2021; Tahroudi et al. 2022; Nazeri Tahroudi et al. 2021, 2022; Dastourani and Nazeri Tahroudi 2022), the correlation values more than 0.4 have been accepted for implementing a copula-based model.

Copula functions have a dependence parameter. In order to create a copula, the dependence parameter of the copula must be calculated. To calculate this parameter, the method of inference functions for margins (IFM) hxas been used. To select the best copula function, the non-parametric values of the empirical copula were compared with the values of the parametric copula. In this evaluation method, the copula that is closer to the empirical copula is introduced as the best copula (Genest and Favre 2007). Comparison of the performance of the studied copulas

Table 3 Results of studied statistics in selecting the best marginal distribution

Parameter	Best marginal distribution	NRMSE (%)	NSE	BIAS	R^2
River flow rate $\left(\frac{m^3}{s}\right)$	GEV	1.7856	0.9962	$-7.53 \times 10^{-5}\,\frac{m^3}{s}$	0.9969
Suspended sediment load $\left(\frac{\text{ton}}{\text{day}}\right)$	GEV	2.2201	0.9941	$0.0022\,\frac{\text{ton}}{\text{day}}$	0.9951

shows that among the 7 studied copulas and their three different rotated states (90, 180, and 270 degrees), the Frank copula has the minimum NRMSE, the maximum NSE, R^2 is close to one and the BIAS is close to zero. Therefore, Frank copula was used to analyze the dependence structure of the studied variables. The results of the studied statistics in selecting the best copulas are presented in Table 4.

Figure 5 also shows the probability contour lines of the joint distribution function for the river flow rate and suspended sediment load data with probabilities of 10 to 90%. Using the joint probabilities occurrence, the river flow rate and suspended sediment load in this station can be determined simultaneously. Also, using this curve and the river flow rate, the suspended sediment load can be estimated with different probabilities.

Univariate return period

In order to analyze the univariate frequency analysis of suspended sediment load for the return period of 2- to 1000-year, generalized extreme value distribution (GEV) was used. The results of univariate frequency analysis of suspended sediment load are presented in Fig. 6. As can be seen, the generalized extreme value distribution fits well with the suspended sediment load data. Also, 95% confidence interval was used to ensure the return period calculations. As can be seen from Fig. 6, the suspended sediment load increases as the joint occurrence probability decrease.

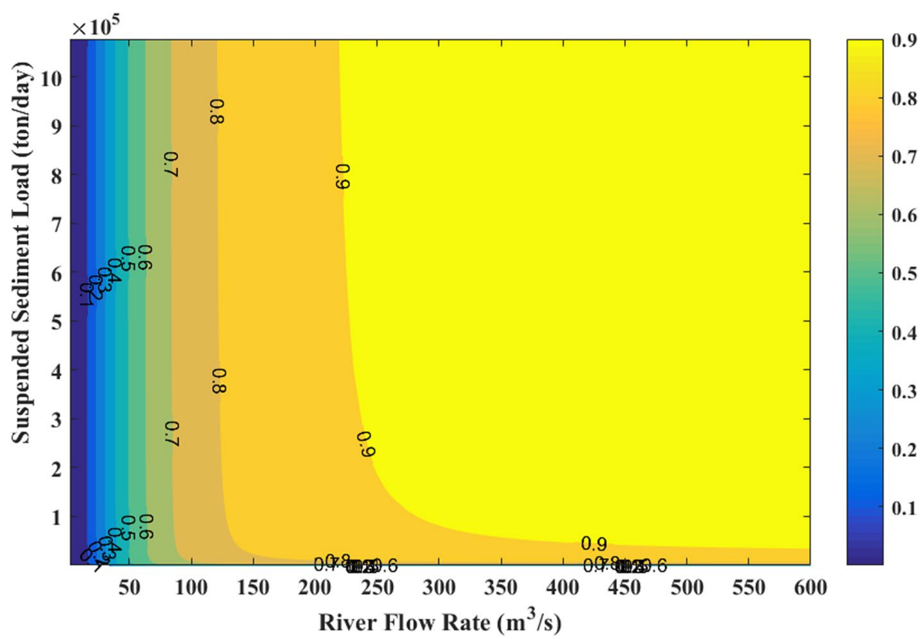
Table 4 Results of the study of the studied statistics for the best copula (Frank copula)

Best copula	NRMSE (%)	NSE	BIAS	R^2
Frank	6.07	0.95	0.05	0.99

Joint return period

Figure 7 shows the joint return periods of the suspended sediment load and river flow rate for the "and" state. According to Fig. 7, it can be seen that with increasing river flow rate and suspended sediment load, the return period also increases. The "and" state means that both variables exceed the threshold. According to Fig. 7, it can be seen that if the river flow rate at Deh Molla hydrometric station is more than 800 m^3/s and the corresponding suspended sediment load is more than 8×10^5 ton/day, the return period will be more than 400 years. Figure 8 shows the joint return periods of the river flow rate and the suspended sediment load for the "or" state. As can be seen, with increasing return period in the "or" state, the river flow rate and suspended sediment load also increase. The "or" state means a state where one of the two variables exceeds the threshold. As can be seen, the joint return period in the "or" state is less than the joint return period in the "and" state. Since the joint return period is inversely related to the joint occurrence probability. In other words, the joint occurrence probability in the "or" state is higher than the joint occurrence probability in the "and" state. Because the probability that each of the variables exceeds the threshold increases and as a result, its return period decreases. According to Fig. 8, it can be seen that in the

Fig. 5 Contour lines of joint occurrence probability of river flow rate and suspended sediment load at Deh Molla hydrometric station



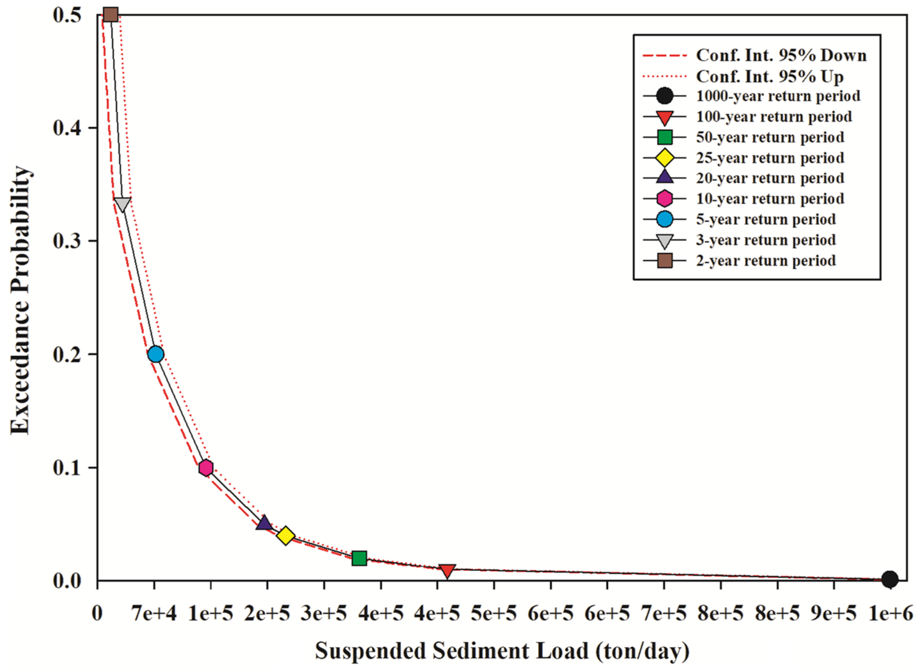


Fig. 6 Univariate estimation of suspended sediment load for the return period of 2- to 1000- year

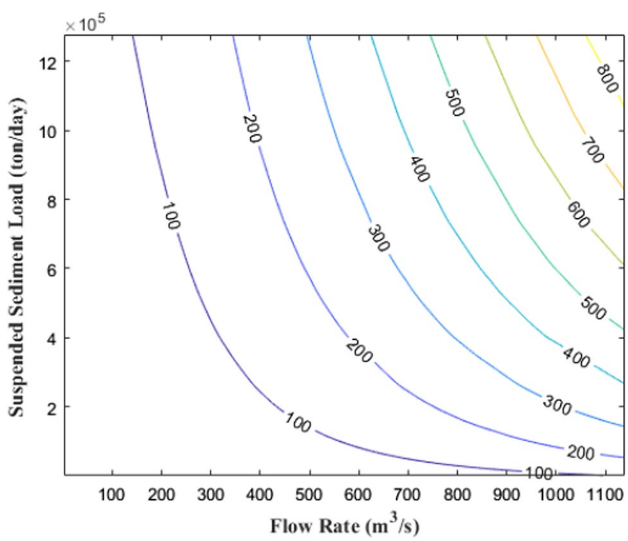


Fig. 7 Joint return period of river flow rate and suspended sediment load in the "and" state

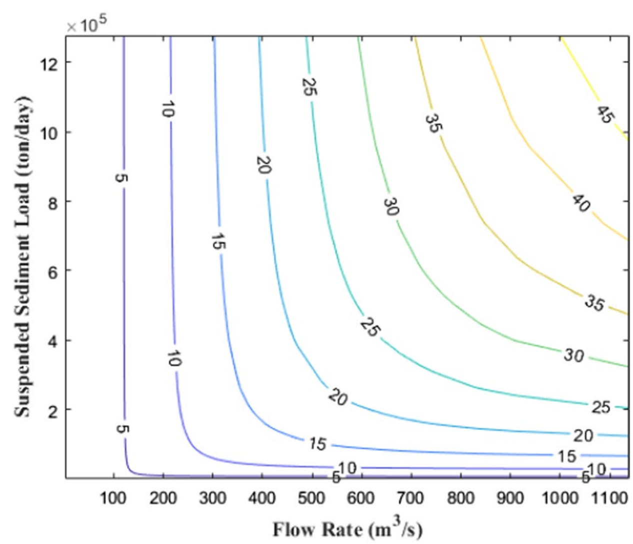


Fig. 8 Joint return period of river flow rate and suspended sediment load in the "or" state

river flow rate of 800 m^3/s, the corresponding suspended sediment load in the return period of 5-, 10-, 15-, 20-, and 25-year is estimated about 0.1×10^5, 0.3×10^5, 1×10^5, 2×10^5, and 3×10^5 ton/day, respectively.

Conditional probability

After determining the best copula function (Frank copula) between the variables, the conditional density of the copula functions was used to calculate the conditional probability. The copula-based conditional density function allows the condition for the occurrence of an independent variable in the dependent variable to be considered. In this case, the presented curves are appropriate to the region and the studied series, and in general, it can be said that the presented curves are specific to that region. In this case, the results of the analysis will be more reliable. Figure 9 shows the conditional probability of suspended sediment load affected by the river flow rate for different return periods. In this study, to investigate the conditional probability, the values

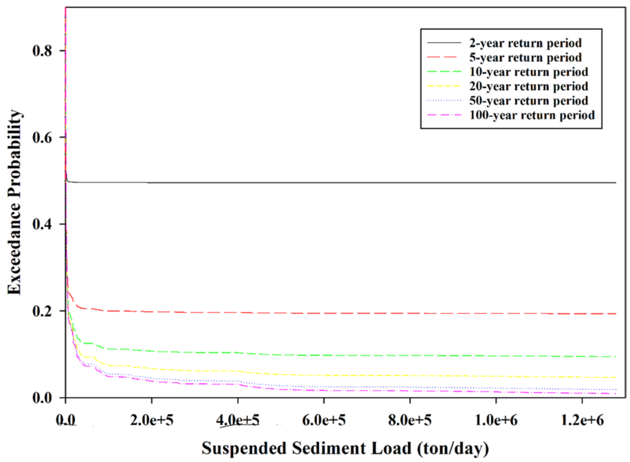


Fig. 9 Conditional probability of suspended sediment load affected by the river flow rate for different return periods

of the calculated quantiles of river flow rate in univariate method were selected as the threshold of river flow rate for 2-, to 100-year return periods. As can be seen, the occurrence probability decreases with increasing return period. And for the return period of more than 5 years, the occurrence probability of suspended sediment load decreases, therefore, the suspended sediment load can be estimated with less probability.

The results of this study, like similar studies in this field, indicated that copulas are an effective tool for multivariate analysis of hydraulic phenomena (Shiau et al. 2021; Li et al. 2020).

Discussion

In this study, at the first step, joint occurrence probability was investigated. This curve can provide useful information for researchers and users about the probabilistic behavior of these two variables. For example, at a river flow rate of 250 m^3/s and a joint occurrence probability of 90%, the suspended sediment load is about 4.29×10^5 ton/day. At a river flow rate of 400 m^3/s and a suspended sediment load of 7.55×10^3 ton/day, the joint occurrence probability is 80%. One of the most important objectives of this study was to provide a typical curve regarding joint occurrence probability. Providing this curve is one of the advantages of the proposed method that can be considered in the basin management. In the next step, the return period was estimated in univariate method and compared with the bivariate method based on the conditional density of copula functions. The results also showed that in the univariate method, the most occurrence probability is 50%. Among the studied return periods, the highest occurrence probability is related to the 2-year return period, which can occur almost every year. The suspended sediment load in the 2-year return period is about 16,527 ton/day. While the average suspended sediment load during the statistical period is about 26,385 ton/day. In the bivariate method, the results were presented as a typical curve. For example, when the river flow rate at the hydrometric station exceeds the threshold of 250 m^3 /s and the suspended sediment load exceeds the threshold of 6.2×10^5 ton/day, the joint return period for the "and" state is 100 years. Also, when the river flow rate exceeds the threshold of 400 m^3/s and the suspended sediment load exceeds the threshold of 10×10^5 ton/day, the joint return period for the "and" state is 200 years. In the case of "and" state, the joint occurrence probability decreases, because both variables are less likely to exceed the threshold, resulting in a higher return period. For example, when the river flow rate exceeds the threshold of 250 m^3/s or the suspended sediment load exceeds the threshold of 6.2×10^5 ton/day, the joint return period for the "or" state is approximately 12 years. Also, when the river flow rate exceeds the threshold of 400 m^3/s or the suspended sediment load approximately exceeds the threshold of 10×10^5 ton/day, the joint return period for the "or" state is 20 years. Finally, the return period was estimated in the bivariate method. Also, in the different return periods, the suspended sediment load varies from approximately 8.2 to 1300×10^3 ton/day. Also, the results showed that in the 2-year return period affected by the river flow rate, the occurrence probability of suspended sediment load at Deh Molla hydrometric station varies from 0.5 to 1. This curve can provide useful and valuable information in the shortest time. For example, the occurrence probability for the suspended sediment load of 2×10^5 ton/day, affected by the river flow rate for the return periods of 2-, 5-, 10-, 20-, 50-, and 100-year, are 49, 19, 10, 6, 4, and 3%, respectively. The proposed method does not have any of the limitations of the data-driven models. In addition, this method has no geographical limitation and does not depend on the characteristics of the basin. The results obtained from this method are specific to the study area and are therefore reliable.

Conclusion

In this study, river flow rate and suspended sediment load data at Deh Molla hydrometric station in Zohreh River were studied. Initially, 11 common marginal distributions including normal, lognormal, exponential, gamma, generalized extreme value, logistics, log logistics, Rayleigh, Nakagami, generalization Pareto, and Weibull were used to perform univariate frequency analysis. The best marginal distribution was selected using NRMSE, NSE, BIAS, and R^2. The generalized extreme value distribution was selected as the best marginal distribution for both variables of river flow rate and suspended sediment load. GEV distribution was used to analyze the univariate frequency of suspended sediment load. In order to analyze the univariate frequency of suspended sediment load for the return period of 2- to 1000-year, a generalized extreme value distribution with a 95% confidence interval was used. The results showed that this distribution has a good fit for the data.

Since copula studies based on the existence of correlations between the studied series, in this study, Kendall's tau correlation test was used to determine the correlation. The results show a positive and significant relationship between river flow rate and suspended sediment load ($\tau = 0.66$). In this study, to create a bivariate distribution, 7 common copula functions including Clayton, Frank, Ali-Mikhail-Haq, Galambos, Farlie-Gumbel-Morgenstern, Plackett, and Gumbel-Hougaard and their rotated states were used. Due to the fact that the Kendall's tau coefficient of the studied variables was not within the allowable range of the Ali-Mikhail-Haq copula, this copula was excluded from the simulation. In order to estimate the dependence parameter of six copula functions, IFM method was used. The best copula function was selected using NRMSE, NSE, R^2, and BIAS. As a result, Frank copula was selected as the best copula for river flow rate and suspended sediment load series.

Then, using the bivariate distribution function obtained from the Frank copula, the joint occurrence probability, the joint return period in the "and" state, the joint return period in the "or" state and the conditional occurrence probability was extracted. The results showed that with increasing the river flow rate and suspended sediment load, the joint occurrence probability and the joint return period increase. The combined return period in the "and" state is longer than the combined return period in the "or" state. As a result, the joint occurrence probability in the "and" state decreases compared to the "or" state, because the probability that both variables exceed the threshold is lower than in the case where one of the two variables exceeds the threshold. Then, the conditional probability of suspended sediment load was studied by considering the river flow rate for different return periods. The results showed that the occurrence probability decreases with increasing return period. Joint analysis of suspended sediment load and river flow rate can be useful in basin management. In fact, a conditional probability diagram can be used to estimate the suspended sediment load for hydrostructures. On the other hand, one of the most important achievements of this study is the presentation of conditional and joint occurrence probability curves that can be used as a typical curve for any hydrometric station. These curves are used to estimate the suspended sediment load affected by the river flow rate at each station with different probabilities and different conditions. The use of copula functions and joint frequency analysis is more detailed than the univariate method. For example, in the 100-year return period, in the univariate method, an amount can be estimated as suspended sediment load, while in the joint method, the amount of suspended sediment load can be estimated with different probabilities. Considering the 66% correlation between the studied variables and the dependence of suspended sediment load on river flow rate, joint frequency analysis is better to be done. In general, hydrological phenomena need to be analyzed in two or more variables due to their interdependence. Because with the increase of effective factors in calculations, the reliability of calculations also increases. Since the proposed method is based on the marginal distribution of data and the conditional density of copula functions, there are no geographical limitation. This method is applicable if there is a correlation between the variables.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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